Hip and Valley Framing II.

By describing a known joint as a mathematical formula and coding this formula as a computer program it would seem possible to "calculate" the joint once, have the computer "remember" the formula, and then ask the computer to crunch out a solution in seconds to any new case. For practical purposes it would also be necessary to show various planes that would be useful views for the shop drawings. When I began in earnest to calculate the formulas for some joints that our shop had already used I began to realize that the available information was incomplete.

Working with a set of angle formulas entitled "Martindale Hip and Valley Roof Angles," intended for the construction of steel hoppers and buildings, I decided to use this resource in my computer program to calculate joints. These angles are derived from the roof pitches, so they are relatively basic design criteria—presumably the roof pitches are known before calculating the joints. When the timber sizes are altered the angles do not change, only the dimensions between points change (so angles made for a stable foundation on which to build my joint formulas. It was clear that some of these Martindale angles were pertinent to timber framing but it was unclear which ones, and to what they applied. In the beginning, I had inferred that it was known where each of the angles was used. On finding out that these angles were actually a puzzle that I had to figure out, I began my work as a mathematician, adding still more complexity to my originally simple idea. I am not by any means a mathematician but I discovered which existing angles were useful and I created and named more as I needed them.

As things progressed I was asked to create one-of-a-kind joints. Because of my familiarity with the angles, I chose to base my joint development work on angles also. I knew that in certain planes a particular angle would be relevant and being able to use this angle made for major short cuts in the development process. In addition to being a drafting machine, the computer is also a calculator and it creates an on-screen test joint. Angles created on the screen can actually be measured and compared with a high degree of accuracy.

While my discussion revolves around developing a joint as a mechanical drawing, in many cases a joint could be developed directly on the stick itself. The drawback to this method is that it is sometimes difficult to tell what is actually happening inside the stick. It is unruly to project one's sight "through" the timber in several planes at once and to imagine how much wood will be left to supply the joint's integrity. Through developing the joint on the screen, it is possible to "engineer" the joint instead of relying on guesswork. Instead of cutting the joint and finding that the tenon narrows so much as to render a peg useless, one can see what the tenon will look like, calculate the load stresses, and modify the joint to suit while still in the design stage.

I have based my joint design on two basic principles:

1. To reduce damage to the mortised member and to increase the integrity of the tenoned member, mortises and tenons should be cut in planes that do not cross the grain of their respective members. Tenons should be aligned with their grain so that all wood fibers contained in the tenon lead, uninterrupted, back to the member. Mortises should interrupt as few fibers as possible—they should be oriented to follow the grain.
2. The acutely-angled sides (usually the load bearing areas) of housings should be cut normal (at right angles) to the side of the mortised member. This prevents sharp-edged, fragile and unstable housings and controls the direction of the loading so that the housing shoulder does not have to resist being split from its timber by the joining timber's acutely-angled knife edge. The load-bearing ends of tenons follow this rule also and become extensions of the housings.

Using the Hawkindale angles shortcuts the development process. These angles relate to various surfaces and planes that exist in a compound joint when looked at from assorted viewpoints. For example, it is simple to construct a normal section of a valley rafter by using the angle C5 to make the backing cut.

Most of the planes that you will be working will usually be the same as those of another joint. If you can relate the two, perhaps you will also see that some of the angles also relate. For example, main R4 describes the line created on the bottom of a valley rafter when it joins the side of a plumb face, say the side of a principal rafter. Relating this plane and the line it makes at the foot of the valley rafter to the peak end, if the valley rafter were to meet the plumb side of a dormer ridge pole the same main R4 would be created, since the two planes are parallel. Adjacent R4 describes the line formed on the bottom of a valley rafter when the upper end of it meets a plumb plane parallel to the main ridge, such as a header.

Using trigonometry, one can create a value for a new angle by using a known angle and "rotating" it by another angle. For example, rotating plan angle D by the valley pitch R1 yields angle R4, while rotating plan angle DD by the main roof pitch SS gives angle P2.

—Rees Acheson

Rees Acheson operated a welding and machine shop from 1972 to 1987, when he joined Benson Woodworking to program the design of compound joinery. Rees is also a successful inventor and founder of the Bortech Corporation.

The Hawkindale Angles©

Definitions, assumptions and limitations. Principal and common rafters are rectangular in section and plumb-sided. Purlins are rectangular with their top surfaces set in the roof plane. Headers are plumb-sided. Hips and valleys are plumb-sided with backing cut into their top surfaces. Ridgepoles (ridge beams, ridge pieces) may take the form of headers or purlins, but in the present Hawkindale system take only the form of headers and are so called. Angles appropriate to purlin-form ridges will be described in a further article.

The centerline of the backing cuts in the top surface of a hip or valley represents the intersection of two roof planes.

Jack rafters and jack purlins are understood to have one end carried by a hip or valley. The term "cripple" is ignored.

All angles are in degrees.

Not all Hawkindale Angles are listed here, but all the listed angles are shown and labelled in the accompanying drawings.

- SS The slope of the main roof.
- S The slope of the adjacent roof (gable, dormer or hip).
- DD The angle in plan between the valley and the main ridge, or between the hip and the main eave.
- D The angle in plan between the valley and the adjacent ridge, or between the hip and the adjacent eave.

- R1 The hip or valley pitch. Taken on the side of the valley or hip, R1 gives the level cut. The plumb cut is then 90 minus R1.
- R2 The angle of a plane perpendicular to the roof surface and parallel to the ridge, projected onto the side of a hip or valley. The side of a purlin is such a plane. R2 is taken on the side of the hip or valley, and 90 minus R2 gives the side shoulder for a jack purlin.
- R4 At the valley peak, adjacent R4 is developed on the bottom surface of the valley by the plumb surface of the header, and main R4 by the plumb surface of the adjacent ridge pole (not shown). At the valley foot, main R4 is the shoulder line on the bottom surface of the valley.
- R5 At the valley peak, main R5 gives the seat housing layout for the bottom surface of the valley on the plumb surface of the header. Adjacent R5 is the angle of this seat housing seen from the end of the header. In general, R5 is the angle made by the valley bottom on any plumb surface.
- R6 At the valley foot, the seat housing angle, projected onto the short (uphill) side of the valley, as viewed from the side of the valley. Adjacent R6 describes connections between main principal rafters or adjacent headers and a valley. Main R6 describes connections between adjacent principal rafters or main headers and a valley.

- P1 Purlin intersection with the side of the valley. Taken on the side of the purlin, 90 minus P1 gives the side cut where the purlin is housed into the valley.
- P2 In the plane of the roof, the angle of intersection between the valley and a principal or jack rafter. The complement 90 minus P2 gives the angle of intersection between the valley and any header, purlin or ridge, again in the plane of the roof.
- P3 On a jack purlin-to-valley joint, the angle of the bottom seat cut, as seen on the short (downhill) side of the purlin.
- P4 The angle of the square end of the tenon and its square shoulder on the long (uphill) side of a jack purlin as seen on the top (roof) surface of the purlin. This angle is also called "the clip angle down the nose." It relates to A8.
- P5 The angle of the square end of the tenon and its square shoulder on the long (downhill) side of a jack rafter as seen on the top (roof) surface of the rafter. This angle relates to A9.
- P6 Jack rafter bottom seat cut viewed on the short side of the rafter. P6 can also be taken on the side of a principal rafter to give the layout of the seat cut for a valley.

- C1 The included angle between a jack purlin and a valley is 90 minus C1, measured normal (perpendicular) to the side of the purlin. Set a saw to 90 minus C1 when cutting the purlin to length from the side.
- C5 The backing angle. Taken on a cross-section of the hip or valley, C5 gives the angle between the bottom of the hip or valley and the roof surface.

- A7 The plumb backing angle. A7 is C5 taken on a plumb section of the hip or valley, or C5 rotated by R1.
- A8 The square shoulder of the long side of a jack purlin projected onto the hip or valley backing. The housing in the valley is cut at 90 minus A8. (Compare P4.)
- A9 The square shoulder on the long side of a jack rafter projected onto the hip or valley backing. The housing in the valley is cut at 90 minus A9. (Compare P5.)

This exposition was edited from material supplied by Rees Acheson and Ed Levin. The computer drawings were executed by Ed Levin using AutoCAD. An earlier article on hip and valley framing appeared in TFN 17.

Hawkindale Angle Formulas©

There are four possible roof conditions.

1. Regular pitch, regular plan. Here the roof slopes are alike (SS = S) and in plan the eaves or ridges meet at right angles (DD = D = 45 degrees).
2. Irregular pitch, regular plan. The roof slopes are different (SS does not equal S) but the eaves still meet at right angles. Here DD is not equal to D. Rather, DD = Arctan (Tan S ÷ Tan SS) and D = Arctan (Tan SS ÷ Tan S).
3. Regular pitch, irregular plan. The roof slopes are alike but the eaves meet at some angle other than 90 degrees. An example would be the hip roof over an octagon. Here the eaves meet at 135 degrees and in plan DD = D = 67.5 degrees.
4. Irregular pitch, irregular plan. The roof slopes differ and the eaves do not meet at right angles. Such a condition would result from a boomerang-shaped building plan with different gable sections on each part.

In the conditions including regular pitch, only one set of the Hawkindale angles is needed. Condition 2 requires different sets of angles for each side of the valley, one for the main roof using SS and DD, and one for the adjacent, using S and D. Condition 4 would call for four sets of angles.

To convert from "over-12" pitches to degrees, divide by 12 and take the Arctan of the result. To reconvert from degrees, take the Tan of the degrees and multiply the result by 12.

The Formulas

R1 = Arctan [Tan S × Sin D]
R2 = Arctan [Sin S × Cos S × Cos D ÷ Tan D]
R4 = Arctan [Tan P2 ÷ Cos C5]
R5 = Arctan [Tan R1 × Cos D]
R6 = Arctan [Tan (90 − R5) ÷ Cos D] - [90 - R1]

P1 = Arctan [Sin S ÷ Tan D]
P2 = Arctan [Cos S ÷ Tan D]
P3 = Arctan [Cos D × Sin R1 × Cos R1 ÷ Cos S]
P4 = P2 - Arctan [Tan R2 × Tan C5 × Cos C5]
P5 = Arctan [Sin D × Cos S ÷ Cos D]
P6 = Arctan [Tan C5 × Cos (90 - P2)]

C1 = Arctan [Sin P1 ÷ Tan S]
C5 = Arctan [Sin R1 ÷ Tan D]

A7 = Arctan [Tan S × Cos D]
A8 = Arctan [Tan R2 × Tan C5 × Cos C5]
A9 = Arctan [Tan R1 × Tan C5 × Cos C5]

Example

Main Roof Pitch (SS	12/12	45.00000 deg
Adj Roof Pitch (S)	18/12	56.30993 deg
Main DD		56.30993 deg
Adjacent D		33.69007 deg

Values in degrees and inches per foot.

Main

R1: 39.76216 10- | R2: 10.47568 2 1/4-

R4: 27.13381 6 1/8+ | R5: 24.77514 5 9/16- | R6: 25.40199 5 11/16+

P1: 25.23940 5 11/16- | P2: 25.23940 5 11/16- | P3: 21.09136 4 5/8

P4: 21.09136 4 5/8 | P5: 46.68614 12 3/4- | P6: 10.30485 2 3/16

C1: 23.09347 5 1/8 | C5: 23.09347 5 1/8

A7: 29.01714 6 11/16- | A8: 4.14804 7/8 | A9: 18.07445 3 15/16-

Adjacent

R1: 39.76216 18- | R2: 29.94344 8 15/16

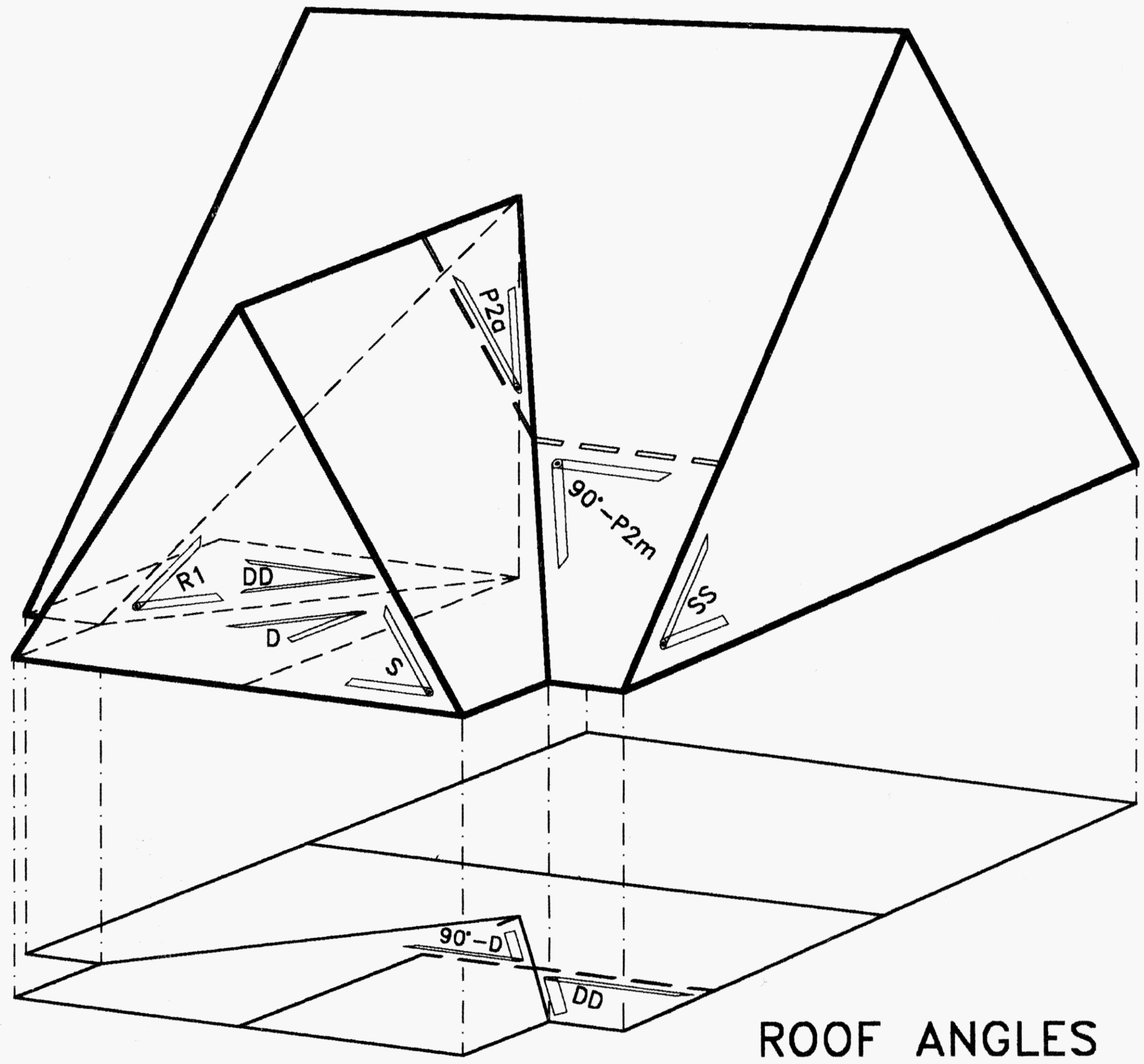
R4: 49.06627 13 13/16+ | R5: 34.69515 8 5/16 | R6: 9.81872 2 1/16+

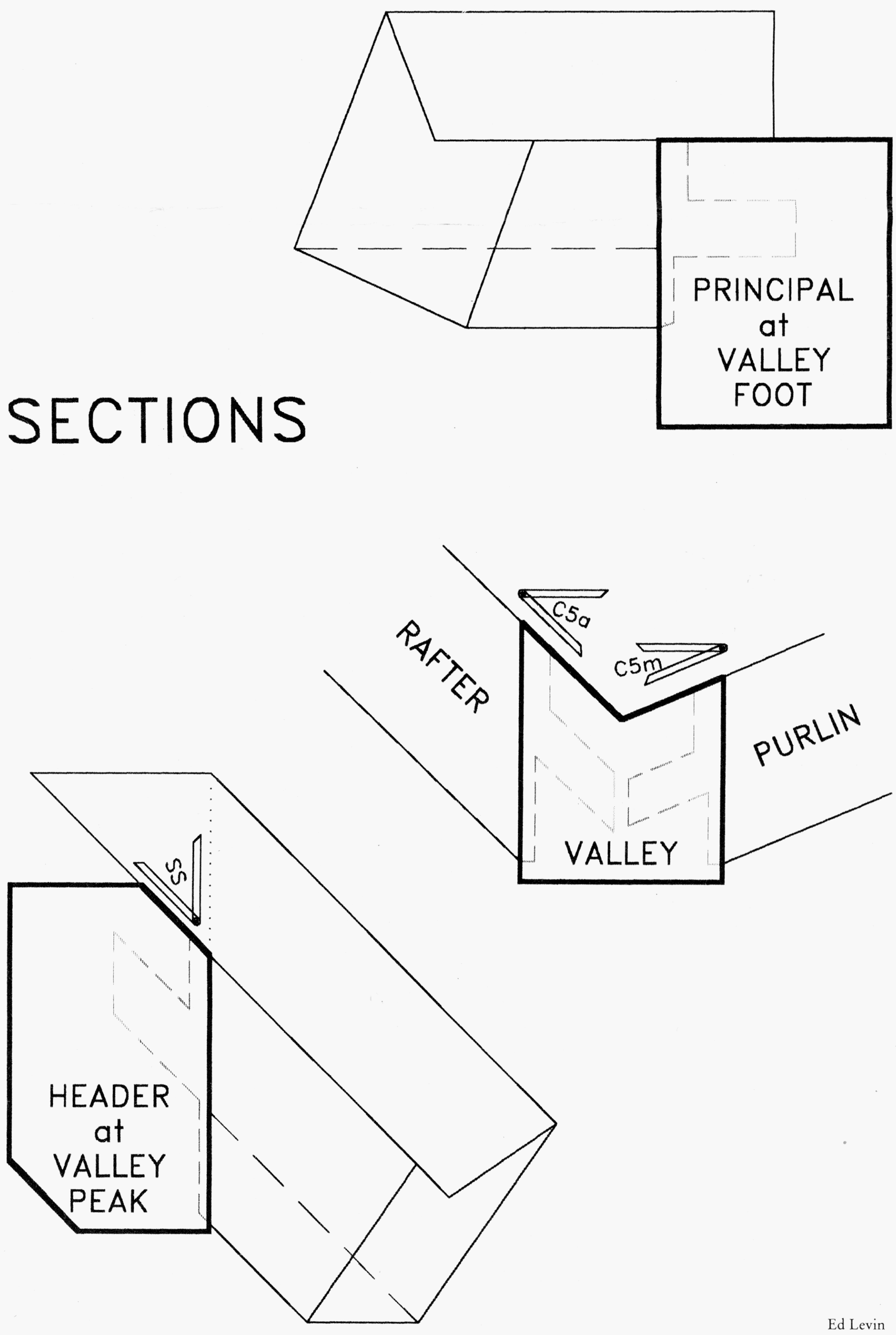
P1: 51.29712 15 - | P2: 39.76216 10- | P3: 36.40874 8 7/8-

P4: 18.02038 3 7/8+ | P5: 20.29440 4 7/16 | P6: 31.53479 7 3/8-

C1: 27.48643 6 1/4 | C5: 43.81306 11 1/2+

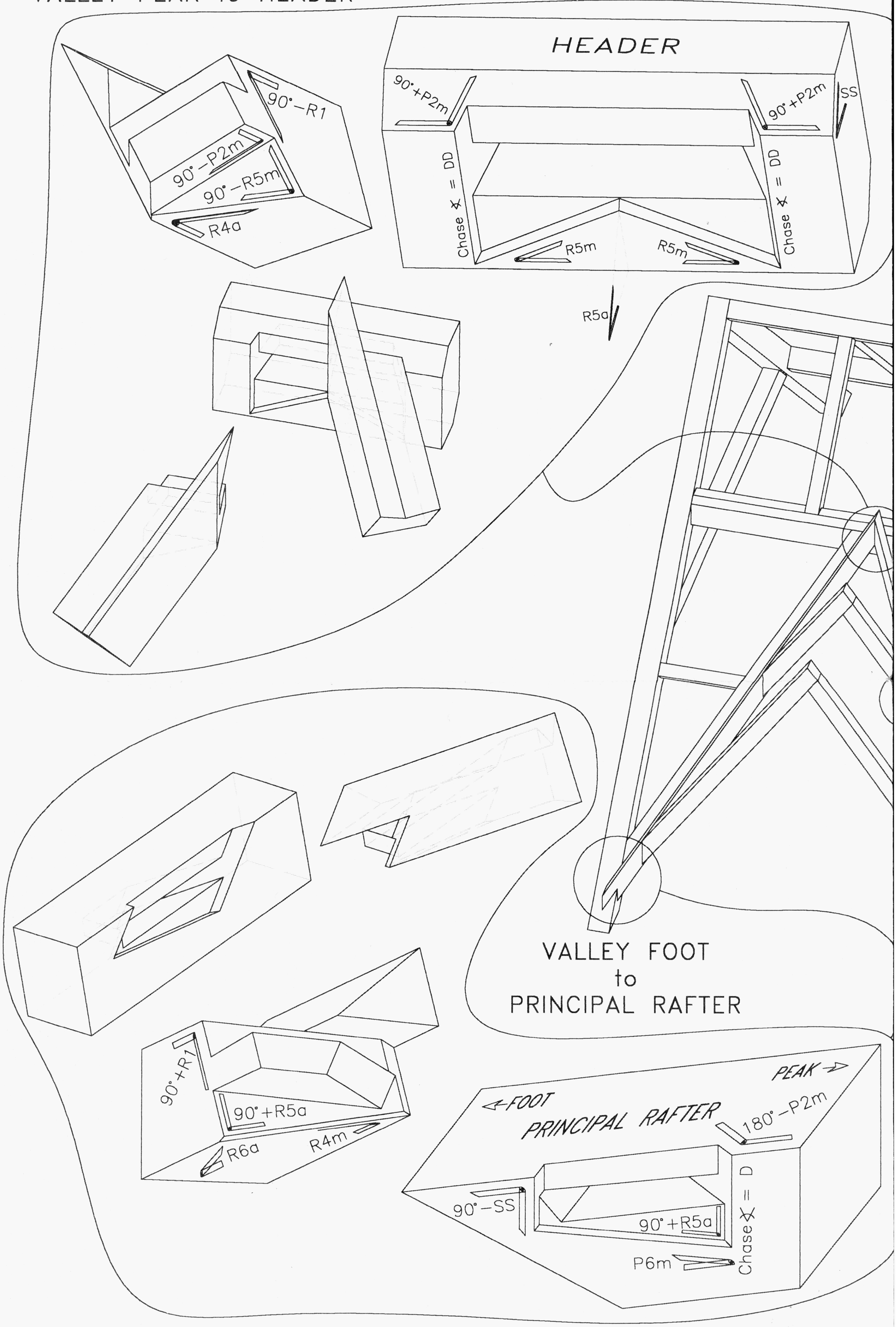
A7: 51.29712 15 - | A8: 21.74178 4 13/16- | A9: 29.94344 6 15/16-



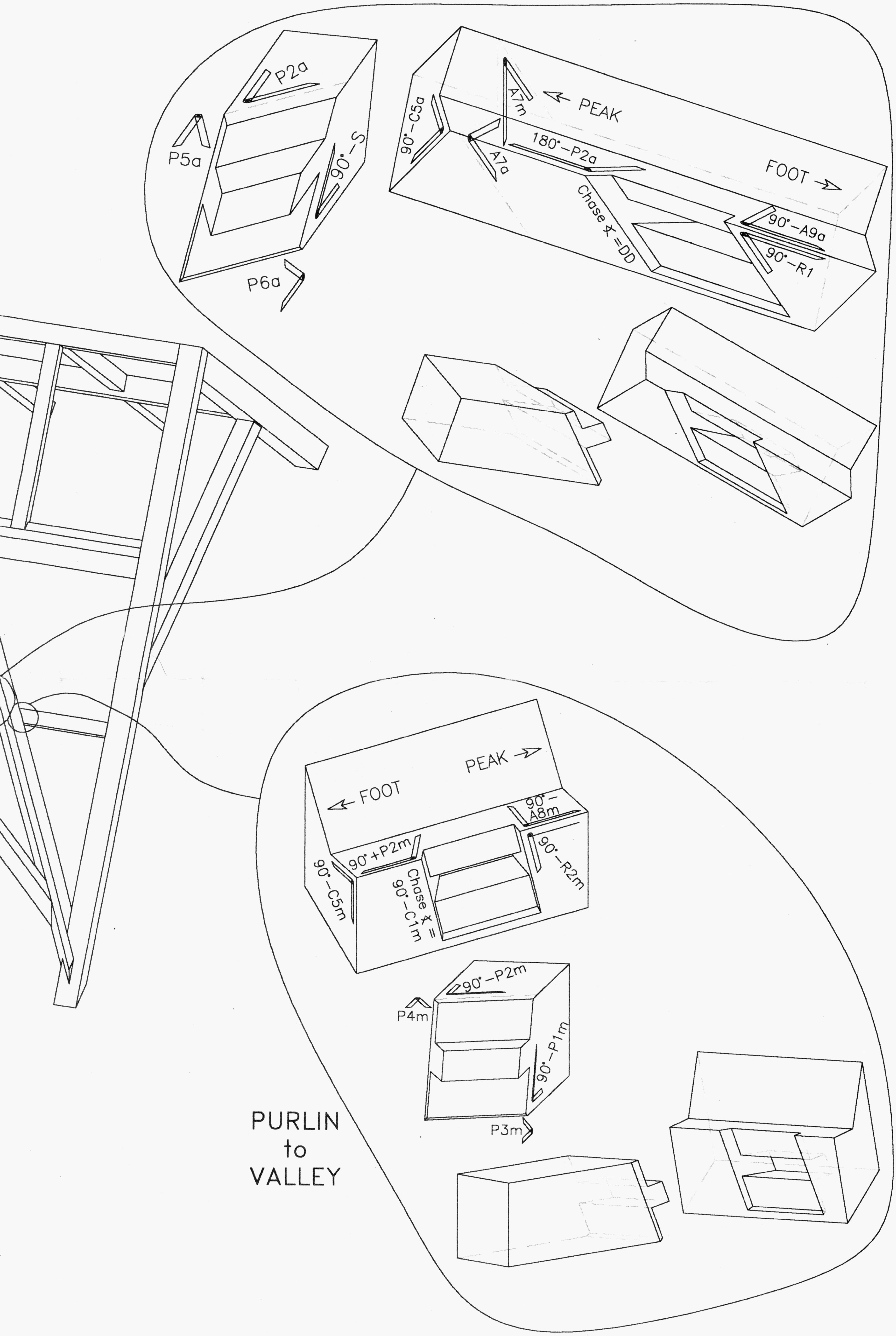


Ed Levin

VALLEY PEAK to HEADER



JACK RAFTER to VALLEY



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