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Title:	Analysis and Design of a King-rod Truss Ring		

Introduction

Photo 1 - King-rod truss roof framing

A king-rod truss, such as that shown in Photo 1, is a common configuration for roof structures, since it makes efficient use of materials (with timber rafters in compression and steel rods in tension), it provides added headroom above the wall plates, and it has aesthetic appeal. Since the two tie rods are pitched upward to provide the added headroom, they impose tension on the vertical king rod. One means of connecting the three tension rods is to use a closed ring, as shown in the photo.

In the design of the ring, if the ratio of the radius of curvature to the depth of the ring (R/d) is less than 5, the flexure formula for straight beams will under-estimate the circumferential stress $\sigma_{\theta\theta}$ in the ring. In such cases, the theory of curved beams should be used for elastic analysis. Also, since the ring is statically indeterminate under the action of the tension rods, the equations of equilibrium alone are insufficient to determine the internal actions in the ring.

The objective of this bulletin is to present a closed-form solution for the internal actions and elastic stresses in a king-post truss ring for a truss subjected to uniformly distributed gravity load (Figure 1). The analysis uses Castigliano's theorem on deflections, and elastic stresses are determined with curved-beam theory. See reference [1] for the development of these methods.

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Notation

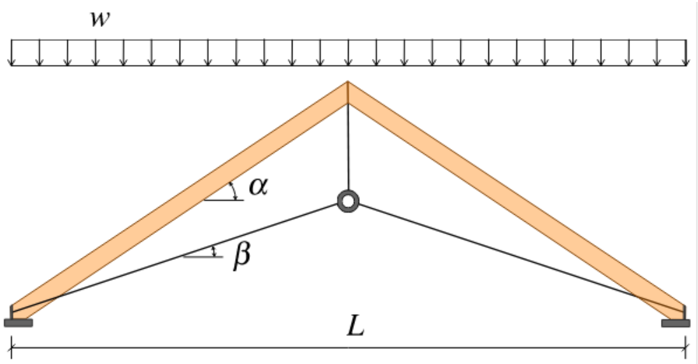

Figure 1 - King-rod truss under uniform gravity load

a = radius to inner edge of ring

b = thickness of the ring

c = radius to outer edge of ring

d = depth of the ring = $c - a$

r = radius from the center of curvature to a point of interest in the cross section of the ring

w = distributed load along the truss span

A = cross sectional area of the ring

A_m = coefficient dependent on the shape of the ring's cross section

E = modulus of elasticity of steel

I_x = moment of inertia of the ring cross section

F_Y = yield stress of steel

K = tension force in the king rod

L = truss span

M_P = plastic moment capacity of a curved beam

M_x, M_0 = bending moment at the cut section and at section A-B, respectively

N, N_0 = normal force at the cut section and at section A-B, respectively

R = radius from the center of curvature to the centroid of the curved-beam cross section

T = tension force in each tie rod

U_M = strain energy in the ring due to bending

V, V_0 = shear force at the cut section and at section A-B respectively

Z = plastic section modulus for the ring

α = rafter pitch

β = tie-rod pitch

γ = angle between the tie rod and the cut section for $\theta > \pi/2 + \beta$

θ = angle from vertical to the cut section

$\sigma_{\theta\theta}$ = circumferential stress in the ring due to bending moment and normal force

Ω = factor of safety for allowable-stress design

Assumptions

1. Plane sections in the ring before loading remain plane after loading.
2. Radial stress and shear stress are sufficiently small that they can be ignored.
3. The radius to depth ratio *R/d* for the ring is greater than 2.0, so we can approximate the strain energy due to bending by the equation for straight beams. Note that the stress distribution in a curved beam is more sensitive to the ratio *R/d* than is the strain energy.
4. The lines of action of the three rods that join the ring are concurrent at the center of the ring and remain so after deflection of the truss under load *w*.

Indeterminate Analysis

Static equilibrium of the truss in Figure 1 under uniformly distributed load *w* gives the tension *T* in the tie rods as

$$T = \frac{wL}{2(\tan\alpha\cos\beta - \sin\beta)} \tag{1}$$

Equilibrium of the ring gives the tension *K* in the king rod (Figure 2) as:

$$K = 2T\sin\beta \tag{2}$$

The ring is symmetric about the vertical axis through the king rod, so we consider only the right half of the ring (Figure 3) by enforcing symmetry conditions on deformation along the symmetry axis. The ring is considered fixed at section *C-D*, and internal actions M_0, N_0 and V_0 act at section *A-B*. Rotation of section *A-B* is restrained by moment M_0, and lateral displacement of section *A-B* is restrained by normal force N_0. By symmetry, $V_0 = K/2$.

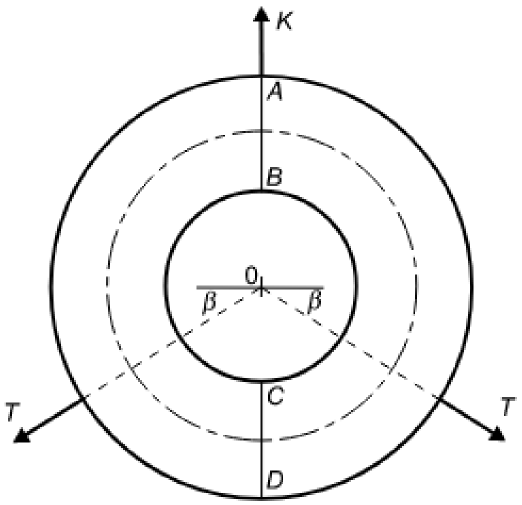

Figure 2 - Free-body diagram of ring

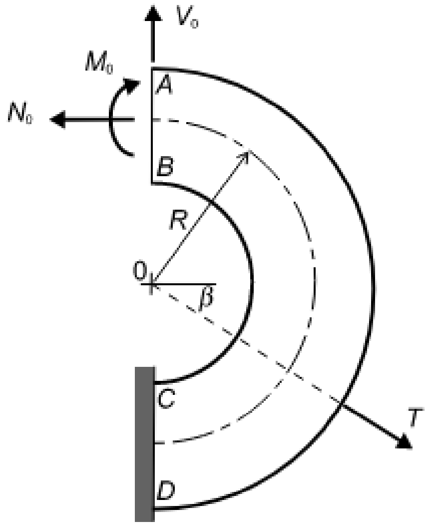

Figure 3 - Right half of ring

With two redundant actions (M_0 and N_0) at section *A-B,* we apply Castigliano's theorem on deflections twice to enforce the boundary conditions. For the rotation of the section due to bending moment

$$\frac{\partial U_M}{\partial M_0} = 0 \tag{3}$$

and for the lateral deflection of the section due to normal force

$$\frac{\partial U_M}{\partial N_0} = 0 \tag{4}$$

where, so long as assumption #3 holds, U_M is given by the equation for straight beams

$$U_M = \int_0^{\pi} \frac{M_x^2}{2EI_x} R\, d\theta \tag{5}$$

The internal actions at a section forming an angle θ with the symmetry axis are determined in two parts, first for $\theta \leq \left(\pi/2 + \beta\right)$ and second for $\theta \geq \left(\pi/2 + \beta\right)$. From the free-body diagram in Figure 4, we have

$$\begin{aligned} N &= N_0 \cos\theta + V_0 \sin\theta \\ V &= -N_0 \sin\theta + V_0 \cos\theta \\ M_x &= M_0 - N_0 R(1 - \cos\theta) + V_0 R \sin\theta \end{aligned} \tag{6}$$

The free-body diagram in Figure 5 gives

$$\begin{aligned} N &= N_0 \cos\theta + V_0 \sin\theta + T \sin\gamma \\ V &= -N_0 \sin\theta + V_0 \cos\theta + T \cos\gamma \\ M_x &= M_0 - N_0 R(1 - \cos\theta) + V_0 R \sin\theta + TR \sin\gamma \end{aligned} \tag{7}$$

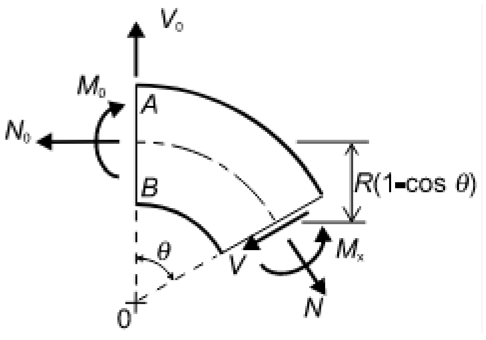

Figure 4 - $\boldsymbol{\theta \leq \left(\pi/2 + \beta\right)}$

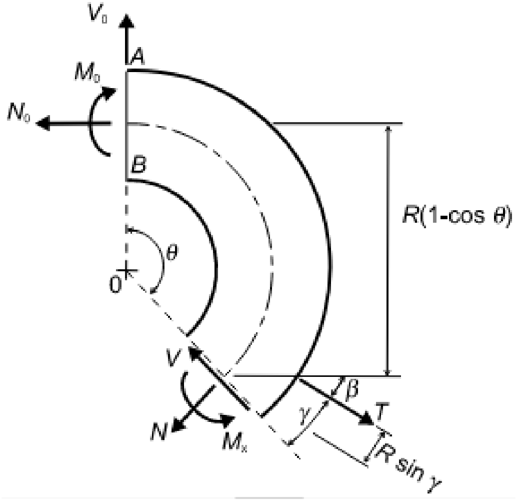

Figure 5 - $\boldsymbol{\theta \geq \left(\pi/2 + \beta\right)}$

Substitution of Eqs. (6) and (7) into Eq. (5) gives the strain energy due to bending in the right half of the ring as

$$U_M = \frac{R}{2EI_x}\int_0^{\frac{\pi}{2}+\beta}[M_0 - N_0R(1-\cos\theta) + V_0R\sin\theta]^2 d\theta$$

$$+\frac{R}{2EI_x}\int_{\frac{\pi}{2}+\beta}^{\pi}[M_0 - N_0R(1-\cos\theta) + V_0R\sin\theta + TR\sin\gamma]^2 d\theta \tag{8}$$

From the boundary condition on rotation at section *A-B* [Eq. (3)], we obtain

$$\frac{\partial U_M}{\partial M_0} = 0 = \frac{R}{EI_x}\int_0^{\pi}[M_0 - N_0R(1-\cos\theta) + V_0R\sin\theta]d\theta$$

$$+\frac{R}{EI_x}\int_{\frac{\pi}{2}+\beta}^{\pi}[TR\sin\gamma]d\theta \tag{9}$$

Integration of Eq. (9), with $\gamma = \theta - \left({}^{\pi}\!/_{2} + \beta\right)$, gives

$$M_0\pi - N_0R\pi + 2V_0R + TR(1-\sin\beta) = 0 \tag{10}$$

from which we obtain the moment at section *A-B* as

$$M_0 = N_0R - \frac{2V_0R}{\pi} - \frac{TR}{\pi}(1-\sin\beta) = 0 \tag{11}$$

We apply the boundary condition on displacement at section *A-B* [Eq. (4)] and get

$$\frac{\partial U_M}{\partial N_0} = 0 = \frac{R}{EI_x}\int_0^{\pi}[-R(1-\cos\theta)][M_0 - N_0R(1-\cos\theta) + V_0R\sin\theta]d\theta$$

$$+\frac{R}{EI_x}\int_{\frac{\pi}{2}+\beta}^{\pi}[-R(1-\cos\theta)][TR\sin\gamma]d\theta \tag{12}$$

Integration of Eq. (12) yields

$$M_0\pi - \frac{3}{2}N_0R\pi + 2V_0R + TR\left[1 - \sin\beta - \left(\frac{\beta}{2} - \frac{\pi}{4}\right)\cos\beta\right] = 0 \tag{13}$$

We subtract Eq. (10) from Eq. (13) and solve for the normal force at section *A-B* to obtain

$$N_0 = T\left(\frac{1}{2} - \frac{\beta}{\pi}\right)\cos\beta \tag{14}$$

Now with M_0 and N_0 given by Eqs. (11) and (14), we can find the internal actions *V*, *N* and M_x at any section in the ring using Eqs. (6) and (7).

Numerical Example

Consider a king-rod truss with span L = 24-ft span, uniformly distributed load w = 480 lb/ft, rafters at an 8/12 pitch and tie rods at a 4/12 pitch (see Figure 1). For the rafters and tie rods,

$$\alpha = 33.69^{\circ} = 0.5880 \text{ rad} \quad \text{and} \quad \beta = 18.435^{\circ} = 0.3218 \text{ rad}$$

Also consider a truss ring that has a rectangular cross section with dimensions: a = 4 in, b = 2 in, and c = 6 in; thus R = 5 in. The steel in the ring is elastic-perfectly plastic with a yield stress of F_Y = 36 ksi.

From Eqs. (1) and (2), we have the forces in the tie rods and king rod as

$$T = 18{,}215 \text{ lb} \quad \text{and} \quad K = 11{,}520 \text{ lb.}$$

With Eqs. (11) and (14), the internal actions at section A-B are,

$$V_0 = K/2 = 5{,}760 \text{ lb}, \quad N_0 = 6{,}870 \text{ lb}, \quad \text{and} \quad M_0 = -3{,}806 \text{ in-lb.}$$

The maximum bending moment in the ring is at the tie-rod location. Substitution of $\theta = \left({}^{\pi}/_{2} + \beta\right)$ into Eq. 6 (or equivalently into Eq. 7 with $\gamma = 0$) gives the normal force and the bending moment as

$$N = 3{,}292 \text{ lb} \quad \text{and} \quad M_x = -21{,}698 \text{ in-lb.}$$

Figure 6 is a plot of bending moment M_x [see Eqs. (6) and (7)] over ring's centroidal line for $\beta = 0.3218$. At the tie-rod attachment, the tie rod causes negative moment, which tends to decrease the radius of curvature. A small negative moment exists at section A-B, where the king rod is attached to the ring. Positive moment develops at section C-D and between section A-B and the tie rod.

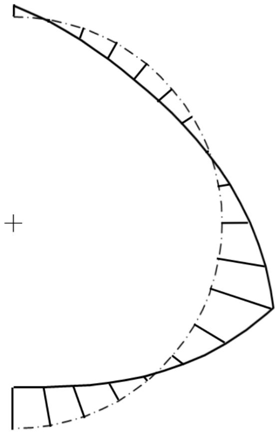

Figure 6 – Moment diagram

Design Based on Elastic Stress Limit

The circumferential stress in a curved beam subjected to normal force and bending moment is given in reference [1] as

$$\sigma_{\theta\theta} = \frac{N}{A} + \frac{M_x(A - rA_m)}{Ar(RA_m - A)} \tag{15}$$

The coefficient A_m depends on the shape of the cross section and is given by

$$A_m = \int \frac{dA}{r} \tag{16}$$

For a rectangular cross section,

$$A_m = b \ln \frac{c}{a} \tag{17}$$

Expressions for A_m for other cross section shapes (triangular, circular, trapezoidal, etc.) are found in reference [1].

The peak values of stress $\sigma_{\theta\theta}$ will occur at either the inner or outer edge of the ring. In the absence of normal force N, the peak stress will be at the inner edge, such as shown in Figure 7. For curved beams with $N = 0$, the neutral axis does not coincide with the centroidal axis of the ring.

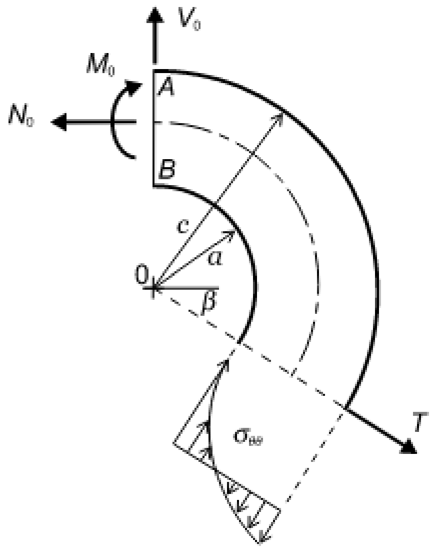

Figure 7 - Circumferential stress for $N = 0$

For the ring in this example, $A = 4$ in^2 and $A_m = 0.811$ in. At the inner edge of the ring, $r = 4$ in, and at the outer edge, $r = 6$ in. So, from Eq. (15), the peak circumferential stresses are

$$\sigma_{\theta\theta\text{-inner}} = -17{,}943 \text{ psi and } \sigma_{\theta\theta\text{-outer}} = 15{,}142 \text{ psi}$$

Note that the contribution of normal force to circumferential stress is $N/A = 823$ psi, which is only around 5% of the stress associated with flexure.

In an allowable-stress approach to design based on first yield, we would limit the circumferential stress to some proportion of the steel yield stress, using an appropriate factor of safety, such as $\Omega = 1.67$.

Design Based on Plastic Capacity

The fully plastic moment capacity M_P in a curved beam under pure bending is the same as that for a straight beam. If the contribution of normal force N on the stress is small, then the plastic capacity of a curved beam is

$$M_P = F_Y Z \tag{18}$$

and for a rectangular cross section,

$$Z = \frac{bd^2}{4} \tag{19}$$

When the circumferential stress due to normal force is significant, the plastic moment capacity must be determined by first locating the plastic neutral axis of the cross section.

In this example, the normal force contribution to circumferential stress is negligible, so with $Z = 2.0$ in^3, the plastic capacity of the ring may be estimated as $M_P = 72{,}000$ in-lb.

If we apply an appropriate factor of safety, such as $\Omega = 2.0$, to the plastic moment capacity and compare that result to M_x, we have another basis for evaluating the design of the ring.

Stress Concentrations

In the above example, design of the tie rod to carry a force $T = 18{,}215$ lb would require a 1.25-in diameter rod of A307 steel, or alternatively a 0.75-in diameter rod of A193-B7 steel. Attachment of the tension rod to the ring could induce stress concentrations that would impact design of the ring. For instance, if the rod were passed through a hole in the ring and secured with a nut on the inside face of the ring, then the circumferential stress on the reduced cross section of the ring would be magnified by the stress concentration around the hole. For instance, a 1-in diameter through-thickness hole in a 2-in wide ring would result in a stress concentration factor of about 2.1 (see Figure 14.16 in reference [1]). So, the peak circumferential stress in the ring would be more than double the average stress across the 1-in net section predicted by Eq. (15).

Note however that in design on the basis of plastic capacity, the effects of stress concentrations are blunted by local yielding, such that the plastic capacity of the net cross section remains unaffected.

Additional Design Considerations

This bulletin focuses on analysis and design of a circular truss ring to quantify and limit circumferential stress $\sigma_{\theta\theta}$ due to internal moment and normal force. Nevertheless, other limit states, not examined here, must also be considered. Some of these limit states might include the following.

- Shear failure of the ring in the vicinity of the tie rods due to high rod tension acting on the gross or net cross-sectional area of the ring.
- Local buckling of the ring at the tie rod connection when fastening hardware, such as a clevis, is used to transfer tie-rod tension into the ring.
- Bearing failure on the inner edge of the ring or at a hole through the ring.
- Bearing or shear failure of fasteners (bolts or pins).
- Elastic or inelastic buckling failure of a thin ring (large d/b ratio).

As with all structural designs, the appropriate list of limit states must be determined based on the particular characteristics of the materials, geometry and loading involved.

Summary

In summary, for a king-rod truss subjected to uniformly distributed load, analysis and design of the ring that secures the three tension rods can be performed by the following steps:

1. Perform static analysis of the truss to determine the reactions at the heels of the rafters and the tensions T and K in the rods (Eqs. 1 and 2).
2. Determine the internal actions V_0, $= K/2$, N_0 from Eq. (14) and M_0 from Eq. (11) at section A-B in the ring.
3. Substitute for V_0, N_0 and M_0 into Eq. (6) and (7) to determine N and M_x at any location θ in the ring.
4. Evaluate either Eq. (6) or (7) for $\theta = \left(\pi/2 + \beta\right)$ to find the maximum value of M_x. and the corresponding value of normal force N.
5. For design based on an elastic stress limit, determine $\sigma_{\theta\theta}$ from Eq. (15) and compare that value to the allowable stress F_Y/Ω. Consider the possible impact of stress concentrations and make any appropriate adjustment to the applied stress $\sigma_{\theta\theta}$.
6. For design based on plastic moment capacity [Eq. (18)], compare M_x at $\theta = \left(\pi/2 + \beta\right)$ to M_P/Ω.

References

[1] Boresi, A. P. and Schmidt, R. J. 2003, *Advanced Mechanics of Materials*, 6E, John Wiley & Sons

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